



Designation: D4105/D4105M – 20

Standard Practice for (Analytical Procedure) for Determining Transmissivity and Storage Coefficient of Nonleaky Confined Aquifers by the Modified Theis Nonequilibrium Method¹

This standard is issued under the fixed designation D4105/D4105M; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

1. Scope*

1.1 This practice covers an analytical procedure for determining transmissivity and storage coefficient of a nonleaky confined aquifer under conditions of radial flow to a fully penetrating well of constant flux. This practice is a shortcut procedure used to apply the Theis nonequilibrium method. The Theis method is described in Practice [D4106](#).

1.2 This practice, along with others, is used in conjunction with the field procedure given in Test Method [D4050](#).

1.3 *Limitations*—The limitations of this practice are primarily related to the correspondence between the field situation and the simplifying assumptions of this practice (see [5.1](#)). Furthermore, application is valid only for values of u less than 0.01 (u is defined in [Eq 2](#), in [8.6](#)).

1.4 All observed and calculated values shall conform to the guidelines for significant digits and rounding established in Practice [D6026](#).

1.4.1 The procedures used to specify how data are collected/recorded or calculated, in this standard are regarded as the industry standard. In addition, they are representative of the significant digits that generally should be retained. The procedures used do not consider material variation, purpose for obtaining the data, special purpose studies, or any considerations for the user's objectives; and it is common practice to increase or reduce significant digits of reported data to be commensurate with these considerations. It is beyond the scope of this standard to consider significant digits used in analytical methods for engineering design.

1.5 *Units*—The values stated in either SI Units or inch-pound units are to be regarded separately as standard. The values in each system may not be exact equivalents; therefore each system shall be used independently of the other. Combining values from the two systems may result in non-

conformance with the standard. Reporting of results in units other than SI shall not be regarded as nonconformance with this practice.

1.6 *This practice offers a set of instructions for performing one or more specific operations. This document cannot replace education or experience and should be used in conjunction with professional judgment. Not all aspects of the practice may be applicable in all circumstances. This ASTM standard is not intended to represent or replace the standard of care by which the adequacy of a given professional service must be judged, nor should this document be applied without the consideration of a project's many unique aspects. The word "Standard" in the title of this document means only that the document has been approved through the ASTM consensus process.*

1.7 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.8 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

2. Referenced Documents

2.1 ASTM Standards:²

[D653 Terminology Relating to Soil, Rock, and Contained Fluids](#)

[D3740 Practice for Minimum Requirements for Agencies Engaged in Testing and/or Inspection of Soil and Rock as Used in Engineering Design and Construction](#)

[D4043 Guide for Selection of Aquifer Test Method in Determining Hydraulic Properties by Well Techniques](#)

¹ This practice is under the jurisdiction of ASTM Committee [D18](#) on Soil and Rock and is the direct responsibility of Subcommittee [D18.21](#) on Groundwater and Vadose Zone Investigations.

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² For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

*A Summary of Changes section appears at the end of this standard



- D4050** Test Method for (Field Procedure) for Withdrawal and Injection Well Testing for Determining Hydraulic Properties of Aquifer Systems
- D4106** Practice for (Analytical Procedure) for Determining Transmissivity and Storage Coefficient of Nonleaky Confined Aquifers by the Theis Nonequilibrium Method
- D6026** Practice for Using Significant Digits in Geotechnical Data

3. Terminology

3.1 Definitions:

3.1.1 For definitions of common technical terms in this standard, refer to Terminology **D653**.

3.2 Symbols and Dimensions:

- 3.2.1 K [LT^{-1}] — hydraulic conductivity.
- 3.2.2 K_{xy} — hydraulic conductivity in the horizontal direction.
- 3.2.3 K_z — hydraulic conductivity in the vertical direction.
- 3.2.4 T [L^2T^{-1}] — transmissivity.
- 3.2.5 S — dimensionless storage coefficient.
- 3.2.6 S_s [L^{-1}] — specific storage.
- 3.2.7 s [L] — drawdown.
- 3.2.8 Q [L^3T^{-1}] — discharge.
- 3.2.9 r [L] — radial distance from control well.
- 3.2.10 t [T] — time.
- 3.2.11 b [L] — thickness of the aquifer.
- 3.2.12 u — dimensionless time parameter.

4. Summary of Practice

4.1 This practice describes an analytical procedure for analyzing data collected during a withdrawal or injection well test. The field procedure (see Test Method **D4050**) involves pumping a control well at a constant rate and measuring the water level response in one or more observation wells or piezometers. The water-level response in the aquifer is a function of the transmissivity and coefficient of storage of the aquifer. Alternatively, the test can be performed by injecting water at a constant rate into the aquifer through the control well. Analysis of buildup of water level in response to injection is similar to analysis of drawdown of water level in response to withdrawal in a confined aquifer. Drawdown of water level is analyzed by plotting drawdown against factors incorporating either time or distance from the control well, or both, and matching the drawdown response with a straight line.

4.2 *Solution* — The solution given by Theis (**1**)³ can be expressed as follows:

$$s = \frac{Q}{4\pi T} \int_u^\infty \frac{e^{-y}}{y} dy \quad (1)$$

where:

$$u = \frac{r^2 S}{4Tt} \quad (2)$$

and:

$$\int_u^\infty \frac{e^{-y}}{y} dy = W(u) = -0.577216 - \log_e u \quad (3)$$

$$+ u - \frac{u^2}{2!2} + \frac{u^3}{3!3} - \frac{u^4}{4!4} + \dots$$

4.3 The sum of the terms to the right of $\log_e u$ in the series of **Eq 3** is not significant when u becomes small.

NOTE 1 — The errors for small values of u , from Kruseman and DeRidder (**1**) are as follows:

Error less than, %:	1	2	5	10
For u smaller than:	0.03	0.05	0.1	0.15

The value of u decreases with increasing time, t , and decreases as the radial distance, r , decreases. Therefore, for large values of t and reasonably small values of r , the terms to the right of $\log_e u$ in **Eq 3** may be neglected as recognized by Theis (**2**) and Jacob (**3**). The Theis equation can then be written as follows:

$$s = \frac{Q}{4\pi T} \left[-0.577216 - \ln \left(r^2 \frac{S}{4Tt} \right) \right] \quad (4)$$

from which it has been shown by Lohman (**4**) that

$$T = \frac{2.3Q}{4\pi \Delta s / \Delta \log_{10} t} \quad (5)$$

and:

$$T = - \frac{2.3Q}{2\pi \Delta s / \Delta \log_{10} r} \quad (6)$$

where:

- $\Delta s / \Delta \log_{10} t$ = the drawdown (measured or projected) over one log cycle of time, and
- $\Delta s / \Delta \log_{10} r$ = the drawdown (measured or projected) over one log cycle of radial distance from the control well.

5. Significance and Use

5.1 Assumptions:

- 5.1.1 Well discharges at a constant rate, Q .
- 5.1.2 Well is of infinitesimal diameter and fully penetrates the aquifer, that is, the well is open to the full thickness of the aquifer.
- 5.1.3 The nonleaky aquifer is homogeneous, isotropic, and areally extensive. A nonleaky aquifer receives insignificant contribution of water from confining beds.
- 5.1.4 Discharge from the well is derived exclusively from storage in the aquifer.
- 5.1.5 The geometry of the assumed aquifer and well conditions are shown in **Fig. 1**.

5.2 Implications of Assumptions:

- 5.2.1 Implicit in the assumptions are the conditions of radial flow. Vertical flow components are induced by a control well that partially penetrates the aquifer, that is, not open to the aquifer through its full thickness. If the control well does not fully penetrate the aquifer, the nearest piezometer or partially penetrating observation well should be located at a distance, r , beyond which vertical flow components are negligible, where according to Reed (**5**)

³ The boldface numbers in parentheses refer to a list of references at the end of this standard.