



Designation: E2578 – 07 (Reapproved 2022)

Standard Practice for Calculation of Mean Sizes/Diameters and Standard Deviations of Particle Size Distributions¹

This standard is issued under the fixed designation E2578; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 The purpose of this practice is to present procedures for calculating mean sizes and standard deviations of size distributions given as histogram data (see Practice E1617). The particle size is assumed to be the diameter of an equivalent sphere, for example, equivalent (area/surface/volume/perimeter) diameter.

1.2 The mean sizes/diameters are defined according to the Moment-Ratio (M-R) definition system.^{2,3,4}

1.3 The values stated in SI units are to be regarded as standard. No other units of measurement are included in this standard.

1.4 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.5 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

¹ This practice is under the jurisdiction of ASTM Committee E56 on Nanotechnology and is the direct responsibility of Subcommittee E56.02 on Physical and Chemical Characterization.

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² Alderliesten, M., “Mean Particle Diameters. Part I: Evaluation of Definition Systems,” *Particle and Particle Systems Characterization*, Vol 7, 1990, pp. 233–241.

³ Alderliesten, M., “Mean Particle Diameters. From Statistical Definition to Physical Understanding,” *Journal of Biopharmaceutical Statistics*, Vol 15, 2005, pp. 295–325.

⁴ Mugele, R.A., and Evans, H.D., “Droplet Size Distribution in Sprays,” *Journal of Industrial and Engineering Chemistry*, Vol 43, 1951, pp. 1317–1324.

2. Referenced Documents

2.1 ASTM Standards:⁵

E1617 Practice for Reporting Particle Size Characterization Data

3. Terminology

3.1 Definitions of Terms Specific to This Standard:

3.1.1 *diameter distribution, n*—the distribution by diameter of particles as a function of their size.

3.1.2 *equivalent diameter, n*—diameter of a circle or sphere which behaves like the observed particle relative to or deduced from a chosen property.

3.1.3 *geometric standard deviation, n*—exponential of the standard deviation of the distribution of log-transformed particle sizes.

3.1.4 *histogram, n*—a diagram of rectangular bars proportional in area to the frequency of particles within the particle size intervals of the bars.

3.1.5 *lognormal distribution, n*—a distribution of particle size, whose logarithm has a normal distribution; the left tail of a lognormal distribution has a steep slope on a linear size scale, whereas the right tail decreases gradually.

3.1.6 *mean particle size/diameter, n*—size or diameter of a hypothetical particle such that a population of particles having that size/diameter has, for a purpose involved, properties which are equal to those of a population of particles with different sizes/diameters and having that size/diameter as a mean size/diameter.

3.1.7 *moment of a distribution, n*—a moment is the mean value of a power of the particle sizes (the 3rd moment is proportional to the mean volume of the particles).

3.1.8 *normal distribution, n*—a distribution which is also known as Gaussian distribution and as bell-shaped curve because the graph of its probability density resembles a bell.

3.1.9 *number distribution, n*—the distribution by number of particles as a function of their size.

⁵ For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

3.1.10 *order of mean diameter*, n —the sum of the subscripts p and q of the mean diameter $\bar{D}_{p,q}$.

3.1.11 *particle*, n —a discrete piece of matter.

3.1.12 *particle diameter/size*, n —some consistent measure of the spatial extent of a particle (see *equivalent diameter*).

3.1.13 *particle size distribution*, n —a description of the size and frequency of particles in a population.

3.1.14 *population*, n —a set of particles concerning which statistical inferences are to be drawn, based on a representative sample taken from the population.

3.1.15 *sample*, n —a part of a population of particles.

3.1.16 *standard deviation*, n —most widely used measure of the width of a frequency distribution.

3.1.17 *surface distribution*, n —the distribution by surface area of particles as a function of their size.

3.1.18 *variance*, n —a measure of spread around the mean; square of the standard deviation.

3.1.19 *volume distribution*, n —the distribution by volume of particles as a function of their size.

4. Summary of Practice

4.1 Samples of particles to be measured should be representative for the population of particles.

4.2 The ‘frequency’ of a particular value of a particle size D can be measured (or expressed) in terms of the number of particles, the cumulated diameters, surfaces or volumes of the particles. The corresponding frequency distributions are called Number, Diameter, Surface, or Volume distributions.

4.3 As class mid points D_i of the histogram intervals the arithmetic mean values of the class boundaries are used.

4.4 Particle shape factors are not taken into account, although their importance in particle size analysis is beyond doubt.

4.5 A coherent nomenclature system is presented which conveys the physical meanings of mean particle diameters.

5. Significance and Use

5.1 Mean particle diameters defined according to the Moment-Ratio (M-R) system are derived from ratios between two moments of a particle size distribution.

6. Mean Particle Sizes/Diameters

6.1 *Moments of Distributions*:

6.1.1 Moments are the basis for defining mean sizes and standard deviations. A random sample, containing N elements from a population of particle sizes D_i , enables estimation of the moments of the size distribution of the population of particle sizes. The r -th sample moment, denoted by M_r' , is defined to be:

$$M_r' = N^{-1} \sum_i n_i D_i^r \quad (1)$$

where $N = \sum_i n_i$, D_i is the midpoint of the i -th interval and n_i is the number of particles in the i -th size class (that is, class frequency). The (arithmetic) sample mean M_1' of the particle

size D is mostly represented by $\bar{D}$. The r -th sample moment about the mean $\bar{D}$, denoted by M_r , is defined by:

$$M_r = N^{-1} \sum_i n_i (D_i - \bar{D})^r \quad (2)$$

6.1.2 The best-known example is the sample variance M_2 . This M_2 always underestimates the population variance σ_D^2 (squared standard deviation). Instead, M_2 multiplied by $N/(N-1)$ is used, which yields an unbiased estimator, s_D^2 , for the population variance. Thus, the sample variance s_D^2 has to be calculated from the equation:

$$s_D^2 = \frac{N}{N-1} M_2 = \frac{\sum_i n_i (D_i - \bar{D})^2}{N-1} \quad (3)$$

6.1.3 Its square root is the standard deviation s_D of the sample (see also 6.3). If the particle sizes D are lognormally distributed, then the logarithm of D , $\ln D$, follows a normal distribution (Gaussian distribution). The geometric mean $\bar{D}_g$ of the particle sizes D equals the exponential of the (arithmetic) mean of the $(\ln D)$ -values:

$$\bar{D}_g = \exp \left[N^{-1} \sum_i n_i (\ln D_i) \right] = \sqrt[N]{\prod_i D_i^{n_i}} \quad (4)$$

6.1.4 The standard deviation $s_{\ln D}$ of the $(\ln D)$ -values can be expressed as:

$$s_{\ln D} = \sqrt{\frac{\sum_i n_i \{ \ln(D_i / \bar{D}_g) \}^2}{N-1}} \quad (5)$$

6.2 *Definition of Mean Diameters $\bar{D}_{p,q}$* :

6.2.1 The mean diameter $\bar{D}_{p,q}$ of a sample of particle sizes is defined as $1/(p-q)$ -th power of the ratio of the p -th and the q -th moment of the Number distribution of the particle sizes:

$$\bar{D}_{p,q} = \left[\frac{M_p'}{M_q'} \right]^{1/(p-q)} \quad \text{if } p \neq q \quad (6)$$

6.2.2 Using Eq 1, Eq 6 can be rewritten as:

$$\bar{D}_{p,q} = \left[\frac{\sum_i n_i D_i^p}{\sum_i n_i D_i^q} \right]^{1/(p-q)} \quad \text{if } p \neq q \quad (7)$$

6.2.3 The powers p and q may have any real value. For equal values of p and q it is possible to derive from Eq 7 that:

$$\bar{D}_{q,q} = \exp \left[\frac{\sum_i n_i D_i^q \ln D_i}{\sum_i n_i D_i^q} \right] \quad \text{if } p = q \quad (8)$$

6.2.4 If $q = 0$, then:

$$\bar{D}_{0,0} = \exp \left[\frac{\sum_i n_i \ln D_i}{\sum_i n_i} \right] = \sqrt[N]{\prod_i D_i^{n_i}} \quad (9)$$

6.2.5 $\bar{D}_{0,0}$ is the well-known geometric mean diameter. The physical dimension of any $\bar{D}_{p,q}$ is equal to that of D itself.